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Mechanisms of vibrational control of heat transfer in a liquid bridge

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Abstract—Convective flows in a liquid bridge subject to axial high frequency vibrations are studied on the base of a generalized Boussinesq approach. The generation of mean vorticity in the dynamical skin-layers near the rigid and free boundaries is taken into account, with the help of effective boundary conditions for the mean components of hydrodynamical fields. The role of viscous damping of surface waves in the generation of the mean flows is analyzed. Numerical calculations are carried out by a finite-difference method. The possibilities of the vibrational control of thermocapillary flow and of the orientation of the surfaces of constant temperature are demonstrated. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

Vibrations have a different influence on the behaviour of fluid systems. If the frequencies are so high that the compressibility effects are important, the vibrations can lead to such a phenomenon as acoustic wind [1]. For low-frequency vibrations different resonance phenomena can be observed, for instance: Faraday ripple [2] or internal waves (see Ref. [3], for example). In the intermediate frequencies range, when the characteristic size of the region is small in comparison with the acoustic wave length but large in comparison with the viscous skin-layer thickness, the viscosity does not have an essential influence on the pulsating field. In this case there is no volumetric generation of the mean flow in the uniform fluid. However, if the fluid density is not constant, for example due to the non-isothermality, the volumetric mechanisms of mean-flow generation do work. This cycle of phenomena called thermovibrational convection has been investigated for a rather long period of time (see the review of Gershuni and Zhukhovitsky [4]). However, most papers consider the behavior of a fluid completely filling a container with rigid walls subject to translational vibrations (i.e. vibrations without change of a container orientation). However, as it was shown by Lyubimov in Ref. [5], this case is exceptional. Indeed, in this case not only mean but also pulsating motion of fluid is absent in the reference frame of the container if the fluid is uniform. Hence, in weakly non-isothermal conditions, thermovibrational convection proves to be a second-order effect. The low values of the pulsating velocity in a proper reference frame makes it possible to use the Boussinesq approximation in which the density variation is taken into account only in volumetric forces. The Boussinesq approach is conventionally used to study buoyancy convection in a static gravitational field. It was extended to the thermovibrational convection in a closed cavity subject to the translational vibrations by Zenkovskaya and Simonenko [6]. However, in the cases of non-uniform vibrations (when a container orientation changes over the period of oscillations, when there is a free surface or fluid interface, or when different boundary domains move according to different laws, as in the case of oscillations of a solid body immersed in a liquid, etc.), the pulsating flow ought to exist even in the absence of temperature differences. In these cases one can expect stronger vibrational convective effects. The validity of the Boussinesq approximation in these situations is not evident *a priori* and needs a special analysis. Such an analysis is carried out in Ref. [5]. It is shown that in the case of non-uniform vibrations the conventional Boussinesq approach cannot be used for the description of thermovibrational convection. In Refs. [7–9] the correct general approach has been worked out for

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NOMENCLATURE

a	amplitude of rod oscillations
a_n, b_n	coefficients of expansion
d	axial length of the ring heater
k_n	$k_n = \pi(1+2n)/2l$
l	aspect ratio, $l = L/R$
l_d	length of viscous damping
$\tilde{p}$	sum of mean pressure and mean density of the kinetic energy of pulsations
r	radial coordinate
t	time
$\vec{u}$	Euler mean flow velocity
u_r, n_z	radial and axial components of the mean velocity
$\vec{u}_L$	Lagrangian mean flow velocity
$\vec{v}_p$	pulsating velocity in the laboratory reference frame
z	axial coordinate
Bi	Biot number, $Bi = \varepsilon\sigma^* T_m^3 R/\chi$
E	dimensionless density of energy of the velocity pulsations, $E = W^2$
F	intermediate function, $F = l + \Sigma_{n=0}^{\infty} b_n$
G_n	intermediate function, $G_n = 1 + We^{-1} k_n(1-k_n^2)(I_1(k_n))/(I_0(k_n))$ $(1+2i\Omega^{-1}k_n^2)/(1-2i\Omega^{-1}k_n^2)$
I_0, I_1	modified Bessel functions of 0th-order and 1st-order
Im	imaginary part of complex number
L	length of the liquid bridge
Ma	Marangoni number, $Ma = \alpha_T' \Delta T R/\rho\nu\chi$
Pr	Prandtl number, $Pr = \nu/\chi$
R	radius of the liquid bridge
Ra_v	vibrational Rayleigh number, $Ra_v = \beta\Delta T(a\omega R)^2/4\nu\chi$
Re	real part of complex number
Re_p	pulsating Reynolds number, $Re_p = a^2\omega/\nu$
$\mathbf{S}$	vector of pulsating transport
T	temperature
$\vec{W}$	non-dimensional complex amplitude of the pulsating velocity
W_r, W_z	radial and axial components of the pulsating velocity
We	Weber number, $We = \rho\omega^2 R^3/\alpha$.

Greek symbols

α	surface tension coefficient
σ_T'	temperature coefficient of surface tension, $\alpha_T' = (\partial\alpha/\partial T)$
β	thermal expansion coefficient
$\beta\Delta T$	Boussinesq parameter
χ	thermal diffusivity
δ	non-dimensional size of the ring heater, $\delta = d/R$
ε	effective heat flux coefficient
ϕ	vorticity
ν	kinematic viscosity
ω	frequency of rod oscillations
ψ	Euler streamfunction
ψ_L	Lagrangian streamfunction
ψ_s	streamfunction of vector field $\mathbf{S}$
ρ	density
σ^*	Stefan–Boltzmann constant
ζ	complex amplitude of the pulsations of the free surface
ΔT	characteristic temperature difference
Ω	dimensionless frequency of rod oscillations
Φ	potential of pulsating velocity.

Subscripts

a	ambient temperature
m	rod temperature
s	quantity related to the pulsating transport.

Superscript

*	complex conjugate.

the description of vibrational flows in a non-isothermal fluid subject to high-frequency vibrations, and a generalized Boussinesq approximation (consistent in the case of non-uniform vibrations) has been developed for weakly non-isothermal conditions. In the present paper we apply the approach developed in Refs. [7–9] to investigate vibrational flows in a liquid zone with a free surface subject to axial high frequency vibrations. It follows from Refs. [7–9] that in this case the isothermal vibrational mechanisms should play an important role in the formation of mean flows, so we start with the consideration of vibrational flows in an isothermal liquid bridge. Vibrational flows in a heated liquid zone and their interaction with the thermocapillary effect, as well as the influence of vibrations on heat transfer, are studied in Section 3.

2. MEAN FLOWS; ISOTHERMAL CASE

Here we consider mean flows in an isothermal cylindrical liquid bridge of length $2L$ and radius R, with a free surface (see Fig. 1). The two rods maintaining the liquid bridge synchronously perform high-frequency oscillations in the direction of the liquid bridge axis. For the simplicity we will neglect the free-surface deformations in average.

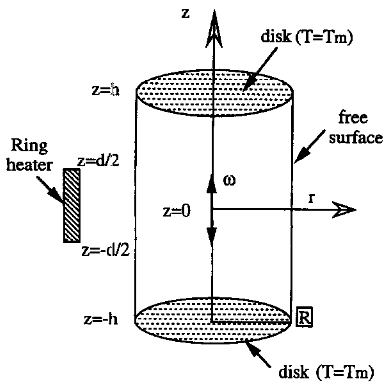

Fig. 1. Problem configuration; coordinates system.

2.1. *Problem formulation*

Let us assume that the frequency of oscillations, ω, is large and the displacement amplitude, a, is small so that the following inequalities are satisfied:

$$(\nu/\omega)^{1/2} \ll L \quad (1)$$

$$a \ll L. \quad (2)$$

Here ν is the kinematic viscosity and L is the characteristic size.

In this case it is possible to decompose the fluid motion into the mean and pulsating parts and, using the multi-scale method, to obtain the governing equations and boundary conditions for these components. For the pulsating velocity field we come to the equations:

$$\vec{W} = \nabla\Phi, \quad \Delta\Phi = 0 \quad (3)$$

where $\vec{W}$ is the non-dimensional complex amplitude of the pulsations velocity in the laboratory reference frame defined in the units $a\omega$. It is related to the pulsating velocity $\vec{v}_p$ by the relation:

$$\vec{v}_p = a\omega[\vec{W}\exp(i\omega t) + \vec{W}^*\exp(-i\omega t)]$$

where * denotes the complex conjugate.

Applying the conventional averaging procedure, taking into account the properties of the pulsating field (3) and the spatial non-uniformity of the pulsating velocity phase we come to the following equations for mean components of hydrodynamical fields [9]:

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\nabla)\vec{u} - \mathbf{S}\times\operatorname{curl}\vec{u} = -\frac{1}{\rho}\nabla\tilde{p} + \nu\Delta\vec{u} \quad (4)$$

$$\operatorname{div}\vec{u} = 0 \quad (5)$$

where $\tilde{p}$, in addition to the usual mean pressure includes the terms of pulsating origin; $\vec{u}$ is the mean velocity; and ρ is the density of the fluid.

Equation (4), valid for the general case of oscillations with non-uniform phase, differs from the equation for mean components of hydrodynamical fields in the case of oscillations with uniform phase (which would coincide with the conventional Navier–Stokes equation) by the presence of the additional term

$$-\mathbf{S}\times\operatorname{curl}\vec{u}$$

where $\mathbf{S}$ is the pulsating transport vector determined by the formulae [10, 11]:

$$\mathbf{S} = \overline{(\vec{q}\nabla)\vec{v}_p}, \quad \frac{\partial\vec{q}}{\partial t} = \vec{v}_p, \quad \bar{\vec{q}} = 0. \quad (6)$$

We use the terminology 'pulsating transport' for the mean transport of vorticity by the pulsations. This effect, found for the first time in Ref. [11] in the study of mass transport by surface wave, takes place only in situations where the pulsating velocity field has non-uniform phase. For this reason pulsating transport is absent in the case of oscillations of the container with rigid walls completely filled with the fluid. However, in the case considered herein the presence of free surface and propagating surface waves leads to the spatial non-uniformity of the oscillations phase, so we need to take pulsating transport effect into account.

Vector $\mathbf{S}$ has a simple physical sense. When a fluid performs a pulsational motion one can consider the average velocity of motion in two different aspects. First, one can consider the average velocity at the fixed point (Euler point of view), in this case the operation of averaging includes velocities of various fluid elements which can be found at a fixed point in some moment of time. It is the average velocity understood so that it forms the vector field $\mathbf{u}$. However, another point of view (the Lagrange one) is possible too, when one follows a fixed fluid element and finds the average value of its velocity: field $\bar{\vec{u}}_L$.

It is evident that the vector fields $\bar{\vec{u}}$ and $\bar{\vec{u}}_L$ differ. It can be shown [9] that the difference $(\bar{\vec{u}}_L - \bar{\vec{u}})$ exactly coincides with the value $\mathbf{S}$.

When an experimentalist observes particles suspended in flow, he sees that they move with the mean velocity $\bar{\vec{u}}_L$. Thus, it is more convenient, from the viewpoint of future comparison with experiments, to present the results in terms of Lagrangian velocity.

In terms of complex amplitude of the pulsating velocity W, the formula for $\mathbf{S}$ can be rewritten in the form:

$$\mathbf{S} = \frac{1}{2}a^2\omega\operatorname{Im}((\vec{W}\nabla)\vec{W}^*) \quad (7)$$

Let us introduce a cylindrical coordinate system in such a way that the values $z = \pm L$ would correspond to the rods and $r = R$ to the free surface (Fig. 1).

We restrict ourselves with axisymmetrical solutions, in which only the radial (W_r, u_r) and axial (W_z, u_z) components of the pulsating and mean velocities differ from zero. Then, it is convenient to introduce the streamfunction, ψ, and the vorticity, ϕ, for the mean flow:

$$u_r = -\frac{1}{r}\frac{\partial\psi}{\partial z}, \quad u_z = \frac{1}{r}\frac{\partial\psi}{\partial r},$$

$$\phi = -\frac{1}{r}\left(\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2}\right).$$

The equations for mean components of hydrodynamical fields being rewritten in terms of ψ and ϕ, in non-dimensional form, are:

$$\frac{\partial\phi}{\partial t} + \frac{1}{r}\left(\frac{\partial(\psi+\psi_s)}{\partial r}\frac{\partial\phi}{\partial z} - \frac{\partial(\psi+\psi_s)}{\partial z}\frac{\partial\phi}{\partial r} + \frac{1}{r}\frac{\partial(\psi+\psi_s)}{\partial z}\phi\right) = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\phi + \frac{\partial^2\phi}{\partial z^2}. \quad (8)$$

Here ψ_s is the streamfunction of the vector field **S** defined by the formula:

$$\psi_s = \frac{1}{2}Re_p \operatorname{Im}\left(r\frac{\partial\Phi}{\partial r}\frac{\partial\Phi^*}{\partial z}\right)$$

where $Re_p = a^2\omega/\nu$ is the pulsating Reynolds number, the quantities, R, R^2/ν, νR are used as the scales for the length, time and streamfunction, respectively.

2.2. *Boundary conditions*

2.2.1. *Boundary conditions for the mean components of hydrodynamical fields.* It was shown in Refs. [11, 12] that the full mass transport vector equals:

$$\rho(\bar{u}+\mathbf{S}). \quad (9)$$

Taking this into account we can formulate the impermeability condition on the rigid and free surfaces in the form:

$$\psi_L = \psi + \psi_s = 0 \quad (10)$$

where ψ_L is the Lagrangian streamfunction.

The potential pulsating field in the interior does not satisfy the no-slip condition on the rigid surfaces. As a result, near these surfaces a thin Stokes layer is formed where the vorticity differs from zero. It was shown by Rayleigh [13] that the non-linear interaction of this pulsating vorticity with the tangential non-uniformities of the pulsating velocity leads to generation of mean vorticity, which can diffuse outside the skin-layer. As it was shown by Schlichting using the boundary-layer-theory methods [14], the above effect can be described with the help of effective boundary conditions for mean velocity on an outer edge of the boundary layer. Applying generalized formula from Ref. [7] we obtain the following condition for the tangential component of mean velocity on the rigid rods, at $z = \pm l = \pm L/R$:

$$\frac{\partial\psi}{\partial z} = \frac{1}{4}Re_p r\left[3\operatorname{Re}\left((1+i)\frac{\partial\Phi}{\partial r}\frac{\partial^2\Phi^*}{\partial r^2}\right) + \frac{2}{r}\left|\frac{\partial\Phi}{\partial r}\right|^2\right]. \quad (11)$$

In the following, this mean-flow generation by high-frequency vibrations, due to the generation of mean vorticity in the dynamical skin-layers near the rigid vibrating boundaries, will be called Schlichting mechanism.

The generation of mean vorticity also takes place near the free surface, where the capillary waves propagate from the rigid rods. The dissipation of the energy of surface waves is accompanied by the decrease of wave momentum. The wave momentum is concentrated in a thin layer near the free surface and the momentum loss is compensated by the mean viscous stress. Owing to this, in the above layer, near the free surface, the mean vorticity generated is of the second order effect with respect to the wave amplitude. This phenomenon was studied for the first time by Longuet-Higgins [11] who described this effect with the help of an effective boundary condition for the shear rate on the free surface. In Ref. [15] the mean flow generated by the waves propagating along the free surface was studied experimentally and theoretically for Czochralsky configuration. In the case considered herein we use an effective boundary condition for the mean vorticity, obtained from the general condition for mean shear-rate tensor formulated in Ref. [7] for an arbitrary wave field:

$$\text{at } r = 1: \phi = 2Re_p\operatorname{Re}\left(\frac{\partial\zeta}{\partial z}\frac{\partial^2\Phi^*}{\partial z^2}\right) \quad (12)$$

where ζ is the complex amplitude of the pulsations of the free surface.

2.2.2. *Boundary conditions for the pulsating fields.* For the pulsating velocity on the rigid rods, at $z = \pm l$ we impose the condition of impermeability:

$$\frac{\partial\Phi}{\partial z} = 1. \quad (13)$$

We should not set any conditions for the tangential components of the pulsating velocity on the rigid rods because of the lowering of the order of the equations for pulsations.

We impose the stuck-edge condition (also called 'fixed contact point' condition) on the contact line, i.e.

$$\zeta = 0 \quad (14)$$

at $z = \pm l$.

Let us discuss the boundary conditions for the pulsating fields on the free surface. Due to different reasons, contrary to the conditions on the rigid rods, we cannot restrict ourselves to consider the limit case of zero viscosity and infinite frequency. First of all, the frequencies used in practice could be comparable with the natural frequencies of the free surface oscillations. Secondly, the limit $\nu \to 0$, $\omega \to \infty$ leads to a singularity in the pulsating field. Thus, we have to consider the non-dimensional frequency of vibrations $\Omega = \omega R^2/\nu$ to be a large but finite quantity. This is not in contradiction with the equations for the pulsating field discussed above, since the pulsational vorticity generated near the free surface decreases exponentially with the increase of distance from this surface and, hence, the influence of the viscosity on the pulsations in the interior can be neglected even in the case of large but finite values of Ω. In addition, to the above mentioned, purely formal, reason (the singularity of the inviscid solution) there are two more reasons to take into account the viscous dissipation of the surface waves. One of these reasons is that in the absence of the dissipation at large but finite frequencies the resonance effects will lead to the infinite growth of the amplitude. The other reason is that taking into account the purpose of this study it is important to consider the mean-flow generation by the surface waves. This generation will take place effectively only if the waves are propagating (i.e. not the standing ones).

In the liquid-bridge configuration the propagating wave system appears due to viscous damping. Thus, formulating the boundary conditions on the free surface, it is principally important to take into account the viscous damping of the surface waves, otherwise the waves propagating from the opposite rods will lead to mutually annihilating mean effects all over the free surface, except near the oscillating rods. Accounting for the viscosity cannot be limited to the consideration of additional terms in a normal stress balance condition. Taking into account the viscosity, but considering the pulsating field in the interior as potential, we have to take into account the vortical component of the velocity in the kinematic condition on the free surface. In Ref. [16], by means of the boundary-layer-theory, a formula was obtained that allows one to express the vortical part of the velocity on the free surface (through the characteristics of the potential pulsating flow) in the case of two-dimensional waves on a flat surface. Generalization of these results for a free-surface curved on average and for an arbitrary wave field was done in Ref. [7]. Applying for the results of Ref. [7] to the case considered herein we come to the boundary conditions for the pulsations on the free surface, at $r = 1$, in the form:

$$i\zeta = \frac{\partial \Phi}{\partial r} - 2i\Omega^{-1}\frac{\partial^3 \Phi}{\partial r \partial z^2} \tag{15}$$

$$i\Phi = 2\Omega^{-1}\frac{\partial^2 \Phi}{\partial z^2} + We^{-1}\left(\zeta + \frac{\partial^2 \zeta}{\partial z^2}\right) \tag{16}$$

where $We = \rho\omega^2 R^3/\alpha$ is the Weber number and α is the surface tension coefficient.

The conditions (15) and (16) are actually the kinematic condition and the condition of normal stress balance for the pulsations accounting for the dissipative effects in the boundary layer near the free surface. The second term in the right-hand-side of (15) is the radial component of the vortical part of the pulsating vorticity field.

To summarize, the problem includes the following dimensionless parameters: pulsating Reynolds number Re_p, geometrical parameter l and the parameters We and Ω.

2.3. *Method of solution*

As one can see from equations (3), (13)–(16), the problem for the pulsating fields is separated and can be solved independently; the structure of the pulsating field is completely defined by the geometry and the vibrations law. This allows one to apply the following algorithm of calculations. We start with the problem for ζ and Φ. To solve this problem, we represent the potential as the sum of a linear function of z and axisymmetrical eigenfunctions of the problem of a liquid zone of infinite extent in the axial direction:

$$\Phi = z + \sum_{n=0}^{\infty} a_n I_0(k_n r)\sin k_n z \tag{17}$$

where $k_n = \pi(1+2n)/2l$ and I_0 is the modified Bessel function of 0th-order.

For representation of the potential (17) the free surface deviation from its equilibrium position is found from the dynamical boundary condition (16) taking into account the stuck-edge condition on the contact line:

$$\zeta = -iWe\left(F\frac{\sin(z)}{\sin(l)} - z - \sum_{n=0}^{\infty} b_n(-1)^n \sin k_n z\right) \tag{18}$$

$$F = l + \sum_{n=0}^{\infty} b_n \tag{19}$$

where the coefficients b_n are connected with a_n by the relation

$$a_n = (-1)^n b_n \frac{1-k_n^2}{I_0(k_n)(1-2i\Omega^{-1}k_n^2)}. \tag{20}$$

Substituting the expansions (17), (18) into the conditions (15) we obtain

$$b_n = \frac{2}{lG_n}\left(\frac{F\cot(l)}{k_n^2-1} - \frac{l}{k_n^2}\right),$$

$$G_n = 1 + We^{-1}k_n(1-k_n^2)\frac{I_1(k_n)}{I_0(k_n)}\frac{1+2i\Omega^{-1}k_n^2}{1-2i\Omega^{-1}k_n^2} \tag{21}$$

where $I_1(k_n)$ is the modified Bessel function of 1st-order.

Equation (21) together with equation (19) form the system of algebraic equations for the coefficients b_n and F. Eliminating b_n from equations (21) and (19) we find for F:

$$F = \frac{l + \sum_{n=0}^{\infty}\beta_n}{1 + \sum_{n=0}^{\infty}\gamma_n} \quad \text{with} \quad \beta_n = -\frac{2}{lG_n k_n^2},$$

$$\gamma_n = -\frac{2\cot(l)}{lG_n(k_n^2-1)}. \tag{22}$$

The infinite series in equation (22) quickly converges. The values of the sums are calculated numerically. The formulae (20)–(22) together with (17) and (18) give the exact solution of the problem (3), (13)–(16) for the pulsating velocity field and the free surface pulsations.

Then, the calculations of the mean fields of the vorticity and streamfunction are carried out by solving equation (8) with boundary conditions (10)–(12) by a finite-difference method at a fixed potential of pulsations. We used an ADI scheme. The Poisson equation for the streamfunction was solved by an iterative method. Most calculations were performed on a 30×60 mesh.

2.4. *Numerical results*

The calculations were carried out at a fixed ratio of zone length to its radius $l = 1$, while the parameters Re_p, We and Ω were varied.

For small values of Re_p (creeping flow regime), the symmetry of the equations and boundary conditions with respect to the plane $z = 0$ leads to the symmetry of the solution. With the increase of Re_p non-linear effects could lead to the break-down of the symmetry through some bifurcation. Nevertheless, the tests performed for some sets of the parameters show that the symmetry of the solutions remains up to large enough parameter values (in the considered range). Thus, it is reasonable to carry out the calculations in half of the zone using the symmetry conditions at $z = 0$.

2.4.1. *Effect of frequency of vibrations.* In the experimental investigation of the mean flows structure the visualization of the flows with the help of light scattering particles is usually used. The mean velocity of particles equals $(u+S)$. Vector $(u+S)$ has simple physical sense: this is the density of mass transport. Thus, it is reasonable to represent the numerical results in terms of the Lagrangian streamfunction ψ_L.

To study the effect of vibrations frequency the calculations were carried out with three non-dimensional parameters: We, Ω and Re_p, varying in a compatible way, such, that the quantities $We^{1/2}/\Omega \equiv On$ and $Re_p/\Omega \equiv (a/R)^2$ remain constant (we take $On^{-2} = 2 \times 10^7$, which corresponds to liquid metals at $R = 1$ cm, and $a/R = 0.01$). The range of the frequency was chosen in such a way that non-dimensional frequency Ω is higher than 10^4 (experimental data on mean-flow generation in the boundary layer prove that only at these values of Ω the high-frequency limit is approached [15]). The values of the vibrations amplitude taken were so small that both the vibrations amplitude itself and the amplitude of surface waves are small in comparison with the characteristic size R. The vibrations frequencies were higher than the natural frequencies of the free surface oscillations at the largest scale, however, the highest resonances could be pronounced if the viscosity is low. This was taken into account in the calculations by controlling the variations of the surface waves amplitude to be inside the range of the validity of theoretical approach (small in comparison with the characteristic size).

Additionally to the resonances damping, the viscosity makes one more important effect on the discussed phenomena. If the viscosity is not too low the free-surface oscillation is not a system of standing waves, but has a significant component corresponding to waves propagating from the rods to the center of the zone. This promotes the effect of the transport of the fluid by the waves. At high viscosity, due to the strong damping of the waves, most of the free surface will be at rest and the surface-streaming effect will be weak. One could expect that the optimal conditions for the mean-flow generation by surface-waves is such a relation of the parameters for which the length of damping l_d of the waves is of the same order of magnitude as the size of the liquid bridge. It is known that for small dissipation, l_d is equal to the ratio of the group velocity to the decrement of damping. For short enough waves the cross-section curvature of the free surface should not be essential; thus, for a very crude estimate we can use the dispersion relation for capillary waves on a flat surface. Then, we obtain: $l_d \approx 3\Omega/4We$.

In Fig. 2(a)–(d) the streamlines of mean flow are presented for four different values of the vibration frequency ($We = 1000, 2200, 4400, 7500$). Mean flow formed by coupled Schlichting and wave mechanisms is of four-vortices structure. The vortices located near the free surface are induced by surface waves propagating along the free-surface from the vibrating rods. The direction of circulation in these vortices is such that the fluid moves out from the rods along the free surface. Two weak symmetrical vortices of Schlichting origin are situated near the two rigid rods. In these vortices the fluid moves along the rigid surfaces in the direction opposite to that of the gradient of pulsating energy, i.e. the flow is clockwise in the lower part of the liquid zone.

As one can see from Fig. 2(a) the contribution of the Schlichting mechanism is the same or even higher than that of the wave mechanism at small values of We (except for resonance zones, narrow enough, where large amplitude of surface waves leads to the sharp growth of the near-surface vortices). At higher values of Weber number, due to the stronger effect of the fluid transport by the waves, the near-surface vortices dominate [Figs. 2(b)–(d)].

Figure 3 shows the dependence of the intensity of mean flow generated by the waves on the Weber number. The maximal value of the Lagrangian streamfunction ψ_L is chosen as characteristic of the flow intensity. As one can see, ψ_L sharply increases near the resonance values (the number of resonance, i.e. the number of waves which can be situated over the zone length can be roughly estimated as $We^{1/3}$). The flow intensity in the areas between the resonances gradually increases with the increase of We, which is related to the decrease of the length of damping (all over the range of the We values, the length of damping

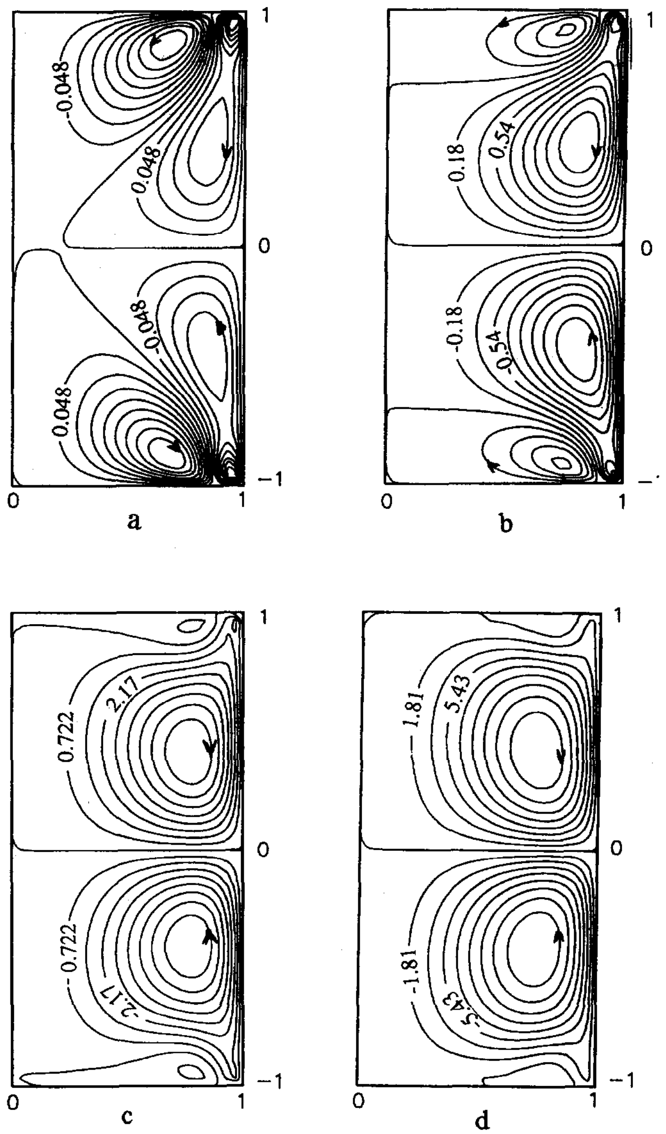

Fig. 2. Streamlines of mean flow induced by coupled Schlichting and wave mechanisms; (a) $We = 1000$, (b) $We = 2200$, (c) $We = 4400$, (d) $We = 7500$.

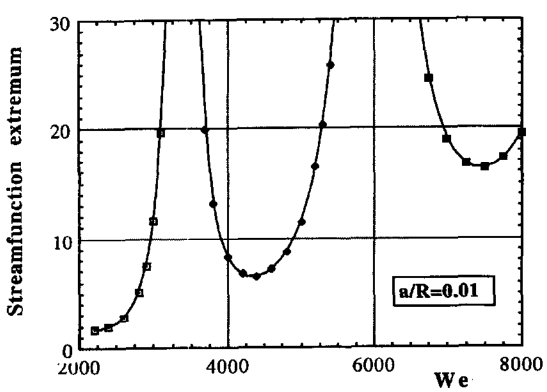

Fig. 3. Intensity of wave-induced flow versus We, including two resonance zones.

is larger than the size of the zone), i.e. with the growth of the propagating component of the waves.

2.4.2. *Effect of the vibration amplitude.* The only dimensionless parameter characterizing the vibration amplitude in isothermal problems is the pulsational Reynolds number. The dependence of the intensity of the mean flow generated by the waves on this parameter is presented in Fig. 4 at a fixed value of vibration frequency ($We = 7500$). As one sees, with the increase of the vibration amplitude, the wave-induced flow intensity increases nearly linearly.

2.4.3. *Concluding remarks.* Thus, the numerical results demonstrate the existence of a parameter range where the resulting flow has the structure of two strongly dominating vortices with the direction of the cir-

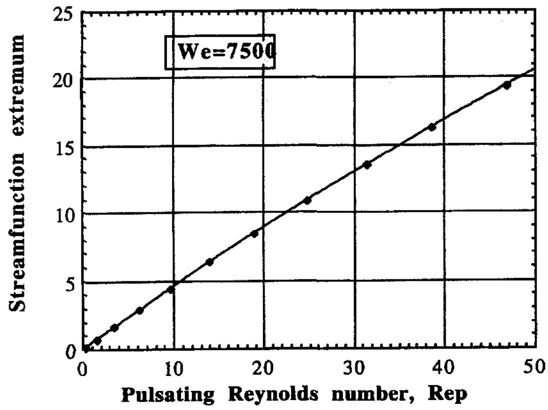

Fig. 4. Intensity of wave-induced flow versus Re_p, for $We = 7500$.

culation opposite to that of thermocapillary flow (in the case of normal thermocapillary effect and for usual heating conditions for the floating zone, i.e. heating of the central part of the zone and cold rods). The localization of these vortices is the same as that of the thermocapillary ones. This means that the vibrational mechanisms can be effectively used for the suppression of thermocapillary flow. The interaction of the vibrational mechanisms with thermocapillary convection is studied in Section. 3.

3. CONVECTIVE FLOWS IN A HEATED LIQUID BRIDGE

This section deals with the flows in non-isothermal fluid subject to non-uniform high-frequency vibrations. In contrast with the situation considered in Section 2, the non-uniformity of density caused by non-isothermal conditions leads to an additional, volumetric, vibrational mechanism of generation of time-averaged flow, namely the thermovibrational convection. We consider the convective flows in a cylindrical liquid zone with the free surface heated with a ring heater. The rods of the zone synchronously perform high-frequency vibrations in the axial direction. In this case three different vibrational mechanisms of time-average flow generation do work. Two of them are isothermal; they are related to the existence of dynamical skin-layers near the rigid rods and the free surface. The third mechanism is volumetric; it is the thermovibrational effect of the first order with respect to the parameter of non-isothermality. We are interested in the interaction and relative role of all vibrational mechanisms, and by the interaction between them and the thermocapillary convection.

Mean flows in a heated liquid zone subject to high-frequency vibrations were considered previously in Refs. [17–22]. For the first time the influence of axial high-frequency vibrations on the flow and heat transfer in the floating zone was studied in Ref. [17] on the basis of the conventional approach; the possibility of partial suppression of thermocapillary flow by vibrations was demonstrated and the orientating effect of vibrations on the surfaces of constant temperature was found. Similar results were obtained on the basis of the same approach in a later work [18]. Later on, we investigated the same problem in the framework of the new approach developed in Refs. [7–9] (see Refs. [19–21]); the new thermovibrational effect of the first order with respect to the Boussinesq parameter and the interaction of this mechanism with the Schlichting's and thermocapillary flow were analyzed. In Ref. [22] surface streaming due to vibrations was studied experimentally for the half zone configuration. The same situation was studied theoretically in Ref. [23] for low-frequency vibrations. In this section we consider the convective flow in a floating zone taking into account both the volumetric thermovibrational effect of the first order and the generation of mean vorticity in boundary layers (near the rigid and free boundaries). The thermocapillary effect is taken into account, whereas the free-surface deformations on average are neglected. Gravity is absent.

3.1. *Problem formulation*

We are interested by the case of weakly non-isothermal conditions, i.e. consider the situation where the relative variations of density due to non-isothermality are small. In this case it is possible to use the equations of thermovibrational convection obtained in Refs. [7–9] for $\beta\Delta T \ll 1$ (where β is the thermal expansion coefficient and ΔT a characteristic temperature difference):

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\nabla)\vec{u} - \mathbf{S}\times \operatorname{curl}\vec{u} - \frac{1}{4}\beta a^2\omega^2 W^2 \nabla T = -\frac{1}{\rho}\nabla\tilde{p} + \nu\Delta\vec{u} \quad (23)$$

$$\frac{\partial T}{\partial t} + (\vec{u}+\mathbf{S})\nabla T = \chi\Delta T \quad (24)$$

$$\operatorname{div}\vec{u} = 0 \quad (25)$$

$$\operatorname{curl}\vec{W} = 0, \quad \operatorname{div}\vec{W} = 0. \quad (26)$$

Here χ is the thermal diffusivity and T is the temperature.

The rods at $z = \pm L$ are assumed to be isothermal:

$$T = T_m. \quad (27)$$

For a free surface we accept the condition of heat exchange with the surroundings:

$$\frac{\partial T}{\partial r} = -h(T - T_a) \quad (28)$$

where h is the effective heat flux coefficient and T_a is the temperature of the surroundings; its dependence on axial coordinate models the heat supplied by a ring heater

$$T_a(z) = T_m + \Delta T \exp(-z^2/d^2) \quad (29)$$

where d is the axial length of the ring heater.

As in Section 2 we consider axisymmetrical solutions in which only the radial and axial components of the pulsating and mean velocities differ from zero. The equations of thermovibrational convection (23)–(26), being rewritten in terms of ψ, ϕ, T and pulsations potential Φ, in non-dimensional form, are:

$$\frac{\partial \phi}{\partial t} + \frac{1}{r}\left(\frac{\partial \psi_L}{\partial r}\frac{\partial \phi}{\partial z} - \frac{\partial \psi_L}{\partial z}\frac{\partial \phi}{\partial r} + \frac{1}{r}\frac{\partial \psi_L}{\partial z}\phi\right) + \frac{Ra_v}{Pr}\left(\frac{\partial E}{\partial r}\frac{\partial T}{\partial z} - \frac{\partial E}{\partial z}\frac{\partial T}{\partial r}\right) = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} - \frac{1}{r^2}\phi + \frac{\partial^2 \phi}{\partial z^2} \quad (30)$$

$$\frac{\partial T}{\partial t} + \frac{1}{r}\left(\frac{\partial \psi_L}{\partial r}\frac{\partial T}{\partial z} - \frac{\partial \psi_L}{\partial z}\frac{\partial T}{\partial r}\right) = \frac{1}{Pr}\Delta T \quad (31)$$

$$\Delta \Phi = 0. \quad (32)$$

Here $E = W^2$ is the dimensionless density of energy of velocity pulsations, $Ra_v = \beta \Delta T (a\omega R)^2/4\nu\chi$ is the vibrational Rayleigh number characterizing the intensity of volumetric generation of mean flows due to the temperature non-uniformity of density, $Pr = \nu/\chi$ is the Prandtl number; the same notations are used for the dimensionless variables.

3.2. *Boundary conditions*

As in the isothermal case we impose for pulsating fields of velocity and free-surface displacement on rigid rods the condition of impermeability (13) and the stuck-edge condition (14), while on the free surface we impose the kinematic condition (15) and the condition of normal-stress balance for pulsations (16), accounting for the dissipative effects in the boundary layer near the free surface. The generation of mean vorticity in the dynamical skin-layers near the rigid rods is taken into account with the help of the effective boundary condition (11) for the tangential component of mean velocity on these surfaces.

The generation of a mean vorticity near the free surface is taken into account with the help of the effective boundary condition for the shear rate on this surface, similar to condition (12). Thus, in the presence of thermocapillary effect the boundary conditions for the mean velocity on the free surface are:

$$\psi_L = 0, \quad \phi = 2Re_p Re\left(\frac{\partial \zeta}{\partial z}\frac{\partial^2 \Phi^*}{\partial z^2}\right) + \frac{Ma}{Pr}\frac{\partial T}{\partial z} \quad (33)$$

where $Ma = \alpha'_T \Delta T R/\rho\nu\chi$ is the Marangoni number, $\alpha'_T = \partial\alpha/\partial T$.

The thermal boundary conditions in non-dimensional form are

$$T = 0 \quad (34)$$

on the rigid rods at $z = \pm l$, and

$$\frac{\partial T}{\partial r} = -Bi(T - T_a), \quad T_a = \exp(-z^2/\delta^2) \quad (35)$$

on the free surface at $r = 1$. Here $Bi = \varepsilon\sigma^* T_m^3 R/\chi$ is the Biot number and σ^* is the Stefan–Boltzmann constant, $\delta = d/R$.

The equations and boundary conditions are written in non-dimensional form. The same scaling quantities as in Section 2 are used for length, velocity and pressure; the value ΔT is taken as the scale for temperature, the value T_m is chosen as reference temperature.

Non-isothermal problems include the following dimensionless parameters: vibrational Rayleigh number Ra_v, Marangoni number Ma, Prandtl number Pr, Biot number Bi, heater parameter δ, and the parameters Re_p, Ω, We and l defined in Section 2.

3.3. *Numerical results*

The calculations are carried out at fixed values of the parameters Pr, Bi, l and δ: $Pr = 0.02$, $Bi = 2$, $l = 1$, $\delta = 0.5$. Parameters Ma, Ra_v, Re_p, We and Ω are varied (as in the isothermal case, the quantity $We^{1/2}/\Omega = On$ is fixed: $On^{-2} = 2 \times 10^7$). The numerical algorithm is the same as the one used in the calculations for the isothermal case (Section 2).

In Fig. 5(a)–(c) the structure of the mean flow generated by joint isothermal and thermovibrational mechanisms is presented for $Ma = 0$, for fixed values of $We = 7500$ and $Ra_v/Re_p = 46.5$ (which corresponds to fixed frequency of vibrations) and three different values of vibration amplitude ($Ra_v = 18$, 112.5, 450). As one can see, in all three cases the structure of the resulting flow is nearly the same. This is related to the fact that, in the isothermal case, at $We = 7500$, the mean flow generated by the waves is dominating and its structure is nearly the same as that of thermovibrational flow [see Fig. 2(d)]. It follows from the results of Section 2 that for smaller values of We a greater contribution of the Schlichting flow could lead to a more complicated flow structure. However, there is a wide range of the parameters values where the resulting flow is of the structure of two vortices with the direction of circulation opposite to that of a thermocapillary flow. Thus, in this range of parameters the vibrations could be efficient for the suppression of thermocapillary flow.

In Figs 6 and 7 numerical results are presented for the convective flows in presence of a thermocapillary effect. Figure 6(a)–(d) illustrates the change of mean-flow structure with the increase of vibrations amplitude ($Ra_v = 18$, 882, 1458, 1800, 2178, 2592) at fixed values of We, Ra_v/Re_p and Ma ($We = 7500$, $Ra_v/Re_p = 46.5$, $Ma = 200$). As one sees, up to the value $Ra_v = 1800$ the structure of the resulting flow is close to the structure of a pure thermocapillary flow [Fig. 6(a)–(c)]. However, the flow intensity sharply decreases in this range of parameters. The flow in Fig. 6(d) has a multi-

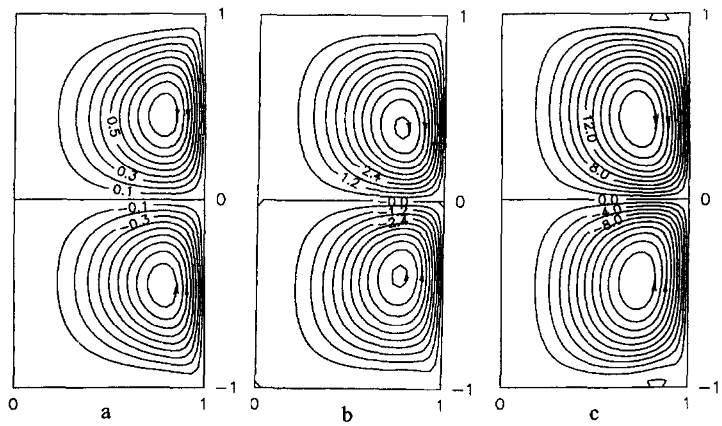

Fig. 5. Streamlines of mean flow for heated liquid bridge with $Ma = 0$; (a) $Ra_v = 18$, (b) $Ra_v = 112.5$, (c) $Ra_v = 450$.

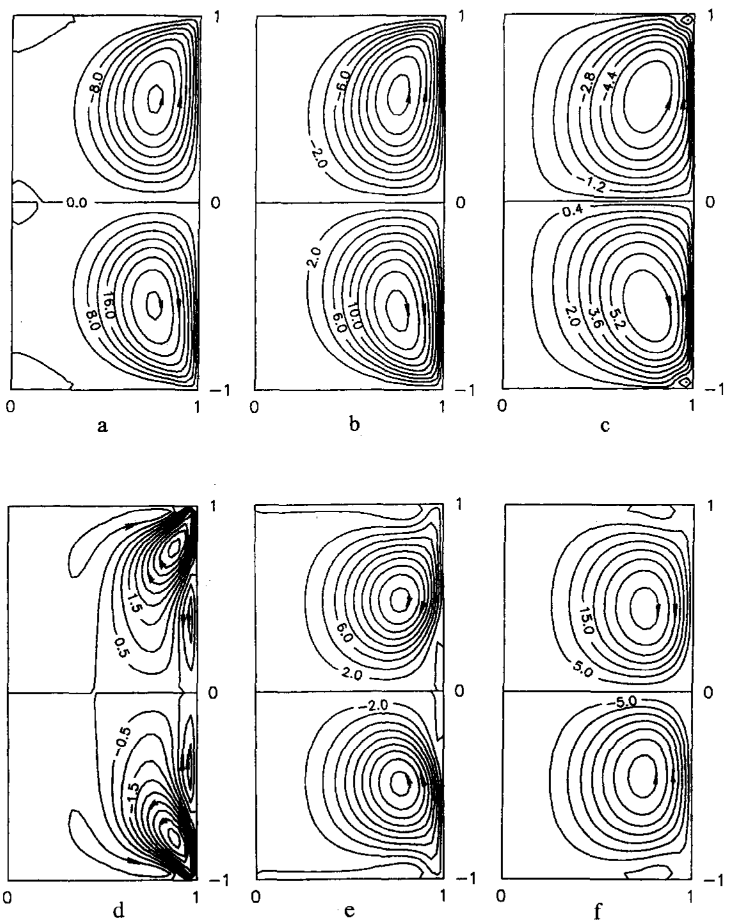

Fig. 6. Streamlines of mean flow for heated liquid bridge, at $Ma = 200$, $We = 7500$ and $Ra_v/Re_p = 46.5$; (a) $Ra_v = 18$, (b) $Ra_v = 882$, (c) $Ra_v = 1458$, (d) $Ra_v = 1800$, (e) $Ra_v = 2178$, (f) $Ra_v = 2592$.

vortex structure, but here all the vortices are rather weak. At even higher values of Ra_v the structure of mean flow is close to the structure of the vibrational flow in the absence of a thermocapillary effect [Fig. 6(e),(f)] (see, Fig. 5 for comparison).

In Fig. 7 the dependencies of total kinetic energy of mean flow on the vibrational Rayleigh number are presented for the same values of *We*, Ra_v/Re_p and two different values of Marangoni number ($Ma = 100, 200$); the values of kinetic energy are related to the values of E_k for pure thermocapillary flow ($Ra_v = 0$). There is a strong effect of suppression of thermocapillary flow due to the coupled effect of two mechanisms: thermovibrational convection and

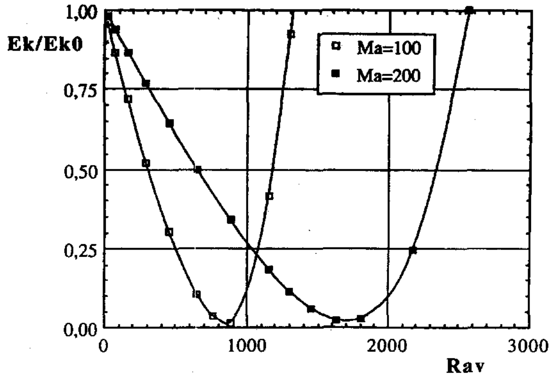

Fig. 7. Total kinetic energy of mean flow versus Ra_v, for $We = 7500$, $Ra_v/Re_p = 46.5$ and two values of Ma.

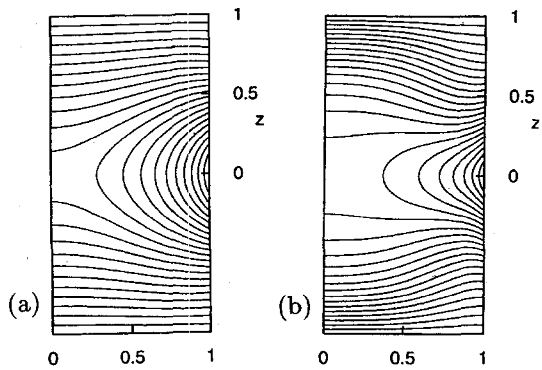

Fig. 8. Isotherms deformation due to vibration, for $We = 7500$, $Ra_v/Re_p = 46.5$; (a) $Ra_v = 9$, (b) $Ra_v = 900$.

surface-wave mechanism. At high values of Ra_v the intensity of the flow induced by vibrations is so high that the total kinetic energy of the mean flow exceeds the value of E_k for pure thermocapillary flow.

Vibrations of high enough intensity make a significant influence on the temperature field. This result was obtained for the first time in our paper [17] where the thermovibrational flows in a floating zone were studied. The calculations carried out by taking into account all the vibrational mechanisms of time-averaged flow generation confirm this conclusion. As one can see from Fig. 8(a) and (b), the deformation of the temperature field by the vibrational flow is such that the lines of constant density (the isotherms in the case considered herein) orientate perpendicularly to the vibrations axis. In technological crystal growth experiments this can result in the flattening of the crystallization front. The orienting effect of the vibrations on the surface shape was found for the first time in our studies for isothermal systems of two immiscible fluids of different densities [25] and the free surface of the liquid [26]. In these cases the vibrations lead to a similar orientation of the interface or the free-surface. It is known that the thermocapillary flow in the case of the floating zone leads just to the opposite result (Figs. 9(a), (b); from Ref. [27]).

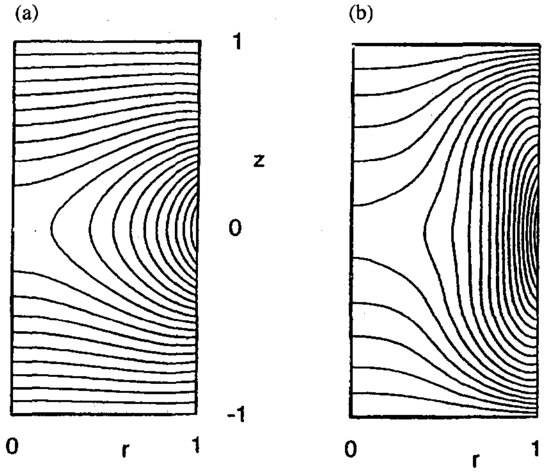

Fig. 9. Isotherms deformation due to thermocapillary flow in the absence of vibrations: (a) $Ma = 10$, (b) $Ma = 500$.

4. CONCLUSION

Convective flows in a cylindrical liquid bridge subject to axial high-frequency vibrations have been studied on the base of a generalized Boussinesq approach. In the isothermal case the mean flows formed by high-frequency vibrations are due to Schlichting and surface-waves mechanisms related to the generation of mean vorticity, in the dynamical skin-layers near the rigid and the free boundaries, respectively. The intensities of mean flows induced by these two mechanisms are governed by the pulsating Reynolds number and the parameters responsible for the capillary effect and viscous damping of surface waves.

In the non-isothermal case the volumetric mechanism of mean flow generation works in addition to the isothermal vibrational mechanisms. This thermovibrational effect is proportional to the parameter of non-isothermality. The interaction of all three vibrational mechanisms and their influence on the thermocapillary flow have been analyzed. The range of governing parameters where the suppression of thermocapillary flow by vibrations takes place has been determined.

The influence of high-frequency vibrations on the temperature field has been studied. It has been found that the vibrations lead to such transformation of the temperature field that the surfaces of constant temperature orientate perpendicularly to the vibrations axis.

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